Goddard Park

Community Primary School

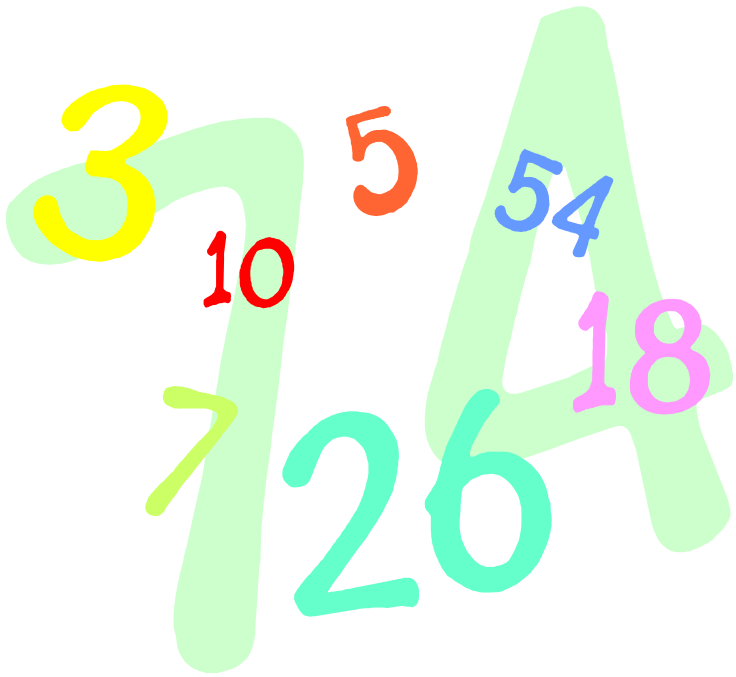
An Integrated Children's Centre

Headteacher: Mike Welsh

Everybody learns, everybody cares

Policy for written methods of calculation using:

The Four Rules



Date of last review: Summer Term '13

Date of Next review: Summer Term '14

Recording methods for Calculations for FS1 to Year 6

This policy has been adapted from the Swindon Primary National Strategy Policy, it has been revised in light of the Goddard Park review of its own Numeracy curriculum. This policy is intended to aid staff in their understanding of the four rules and to help provide progression in informal and formal written calculations. It sets out the range of models and images that staff could use to develop calculation from FS2 to Year 3, alongside the more formal written method that are introduced in KS2.

This policy recognises that that pupils' mental number knowledge and skills are of prime importance. The content of this policy is to give guidance regarding progression with the majority of pupils. It does not set out the exact system of progression that every child must go through.

It is expected that addition/subtraction and multiplication/division, will be taught alongside each other so that pupils can use the relationship between them. Pupils should be encouraged to recognise multiplication as repeated addition and division as repeated subtraction. It is essential that pupils are taught to estimate their answers first and check their calculations with a variety of strategies including the inverse.

Aims and Rationale

Children are introduced to the processes of calculation through practical, oral and mental activities. As children begin to understand the underlying ideas, they develop ways of:-

- Recording to support their thinking and calculation methods
- Using particular methods that apply to special cases
- Interpreting and using the signs and symbols involved

As children's methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and lead to efficient written methods that can be used more generally.

Early practical, oral and mental work *must lay the foundations* by providing children with a good understanding of:-

- How the four operations build on efficient counting strategies
- Place value
- Number facts

(Extract from guidance paper 'Methods of Calculation', The Primary Strategy Renewed Framework, 2006)

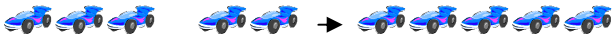
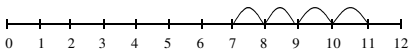
Recording Method

Foundation	Year 1	Year 2	Year 3
DRAWING PICTURES AND MARKS			
	USING SIGNS AND SYMBOLS		
	DRAWING NUMBER LINES		
		USING INFORMAL JOTTINGS	
			EXPLAINING IN WORDS

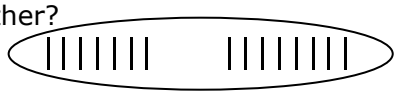
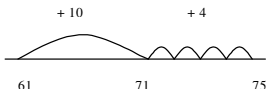
REAL LIFE CONTEXT

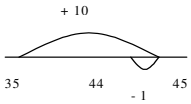
Developing into more formal written methods in KS2.

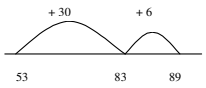
RECORDING ADDITION

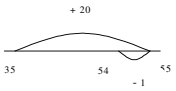
Foundation stage PRSN – Overviews 2, 8 and 9	Year 1
Pictures / marks There are 3 cars in the garage. 2 more arrive. What is the **total**?  Using 2 different colours of lego, build 2 towers. How many bricks are there **altogether**?	**Pictures / marks** Lisa has 5 lollies and Tim has 2 lollies. How many lollies do they have **altogether**?  Using materials to add up
Signs and symbols *(Number sentences modelled by adults)* Use number and photo displays to organise the learning environment.	**Signs and symbols –**Record own number sentences, linked to a practical example 3 + 2 = □ □ = 3 + 2 3 + □ = 5 5 = □ + 2 □ + 2 = 5 5 = 3 + □ □ + □ = 5 5 = □ + □ Record a partitioned number as a number sentence Eg 57= 50 + 7
Number lines *(Prepared number tracks and lines used)* Reinforce counting on a number track e.g how far down the number track can you throw a beanbag?	**Number lines** (numbered) Counting on using a number line 7 + 4  Leading to counting on mentally, keeping the largest number in your head. Recording by - drawing jumps on prepared lines and then constructing own lines
Informal jottings Drawing adding pictures e.g Can the children make 10 in a variety of ways? Use of number staircases How do children find a total?	**Informal jottings** 'Can you put something on paper to show that….?' *(Teachers model jottings appropriate for larger numbers)* Use diagrams to solve problems involving addition
Practical examples • Counting real objects – In construction and small world play, provide plans for models showing numbers eg 10 pieces of lego • Begin to use number lines and count on.	**Practical examples** • Record coins which might be used to 'pay' and 'give change' • Record spots on blank dominoes to show addition sentences • Use hundred square to add multiples of ten • Find different ways of putting 11 spots on three ladybirds.

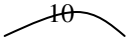
RECORDING ADDITION

<table>
<tr><td>Year 2</td><td>Year 3 (see KS2 Calculation policy also)</td></tr>
<tr><td><u>Pictures / marks</u>
There are 7 people on the bus. 8 more get on. How many people are on the bus altogether?
  </td><td><u>Pictures / marks</u>
As Year 2
Numbers are often too large for pictures to be efficient but pictures/diagrams will continue to be used where appropriate</td></tr>
<tr><td><u>Signs and symbols</u>
5 + 4 = □ □ = 5 + 4
5 + □ = 9 9 = □ + 4
□ + 4 = 9 9 = 5 + □
□ + □ = 9 9 = □ + □
Adding three numbers 1 + □ + 5 = 17
Extend to 14 + 5 = 10 + □
Record a partitioned number as a number sentence eg 53= 40+13 or 30+23</td><td><u>Signs and symbol</u>
13 + 6 = □ □ = 13 + 6
13 + □ = 19 19 = □ + 6
□ + 6 = 19 19 = 13 + □
□ + □ = 19 19 = □ + □
Adding three or more numbers
14 + □ + 6 = 37
Extend to 21 + 6 = □ + 10</td></tr>
<tr><td><u>Number lines</u>
empty)
 

61 + 14
(partition)
 

35 + 9
add 9 by adjusting</td><td><u>Number lines</u>
(numbered)
 

53 + 36
(partition)
 

35 + 19
add 19 by adjusting</td></tr>
<tr><td><u>Informal jottings</u>
<u>Partition into tens and ones and recombine</u>
23 + 12 = 23 + 10 + 2
= 33 + 2
= 35
 

3 + 8 + 7 = 10 + 8
10 + 8= 18
<u>Pairs totalling ten</u>
Partition the 2nd number and add</td><td><u>Other jottings</u>
Partition second no. and add
<u>Partition into tens and ones and recombine</u>
42 + 27 = 42 + 20 + 7
= 62 + 7
= 69
<u>Pairs totalling ten or twenty</u>
4 + 8 + 16 + 2 = 20 + 10
= 30</td></tr>
<tr><td><u>Explaining in words</u>
(Explaining methods and reasoning orally)</td><td><u>Explaining in words</u>
To add 23 and 17 I added 23 and 7 to make 30 and added 10 more to total 40</td></tr>
</table>

NB
It is preferable to only partition the smaller number, as this can then also be applied to subtraction

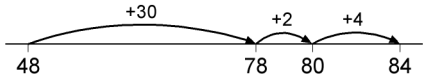
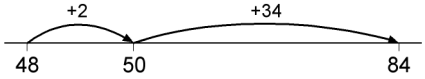
Written methods for addition of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for addition which they know they can rely on when mental methods are not appropriate. These notes show the stages in building up to using an efficient written method for addition of whole numbers by the end of Year 4.

To add successfully, children need to be able to:

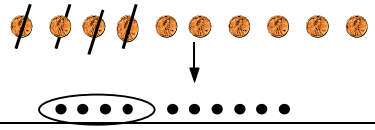
- recall all addition pairs to 9 + 9 and complements in 10;
- add mentally a series of one-digit numbers, such as 5 + 8 + 4;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.

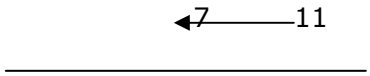
Stage 1: The empty number line • The mental methods that lead to column addition generally involve partitioning, e.g. adding the tens and ones separately, often starting with the tens. Children need to be able to partition numbers in ways other than into tens and ones to help them make multiples of ten by adding in steps. • The empty number line helps to record the steps on the way to calculating the total.	**Stage 1** Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10. 8 + 7 = 15  48 + 36 = 84  or: 
Stage 2: Partitioning • The next stage is to record mental methods using partitioning. Add the tens and then the ones to form partial sums and then add these partial sums. • Partitioning both numbers into tens and ones mirrors the column method where ones are placed under ones and tens under tens. This also links to mental methods.	**Stage 2** Record steps in addition using partitioning: 47 + 76 = 47 + 70 + 6 = 117 + 6 = 123 Partitioned numbers are then written under one another: 47 = 40 + 7 +76 70 + 6 110 + 13 = 123

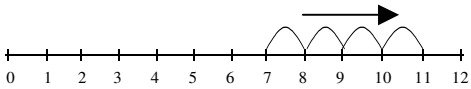
Stage 3: Expanded method in columns • Move on to a layout showing the addition of the tens to the tens and the ones to the ones separately. To find the partial sums either the tens or the ones can be added first, and the total of the partial sums can be found by adding them in any order. • The addition of the tens in the calculation 47 + 76 is described in the words 'forty plus seventy equals one hundred and ten', stressing the link to the related fact 'four plus seven equals eleven'. • The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and in their understanding of place value.	**Stage 3** Write the numbers in columns. Adding the ones first: 47 + 76 13 110 123 368 +493 11 150 700 861
Stage 4: Column method • In this method, recording is reduced further. Carry digits are recorded below the line, using the words 'carry ten' or 'carry one hundred', not 'carry one'. • Later, extend to adding three two-digit numbers, two three-digit numbers and numbers with different numbers of digits.	**Stage 4 (by the end of Y4)** 47 + 76 = 123 (11) 258 + 87 = 345 (11) 366 + 458 = 824 (11) Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable.

RECORDING SUBTRACTION

<table>
<tr><td>Foundation
PRSN – Overviews 2 and 9</td><td>Year 1</td></tr>
<tr><td><u>Pictures / mark</u> We made 6 cakes and ate 2. How many are left?
  </td><td><u>Pictures / marks</u>
Sam spent 4p. What was his change from 10p?
  </td></tr>
<tr><td><u>Signs and symbols</u>
(Number sentences modelled by adults)</td><td><u>Signs and symbols</u>
5 - 2 = □
5 - □ = 3
□ - 2 = 3
□ - □ = 3

□ = 5 - 2
3 = □ - 2
3 = 5 - □
3 = □ - □
Children understand that the order of numbers in a subtraction calculation matters.</td></tr>
<tr><td><u>Number lines</u>

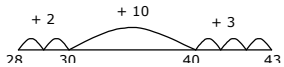
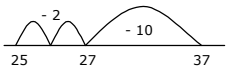
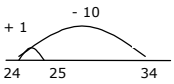
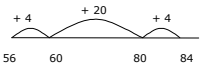
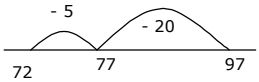
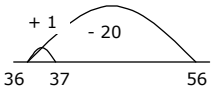
(Prepared number tracks and lines used)</td><td><u>Number lines</u>
Example:- 11-7
Counting back
 

Counting on
the difference between 7 and 11
 

Recording by - drawing jumps on prepared lines
- constructing own lines
Finding the difference using cubes / materials / toys etc and recording as a number sentence</td></tr>
<tr><td><u>Informal jottings</u></td><td><u>Informal jottings</u>

(Teachers model jottings appropriate for larger numbers)
Use diagrams to solve problems involving subtraction</td></tr>
<tr><td><u>Practical Examples</u>
• Finding one less than (numbers to 10)
• Picture/story boards of calculations
• Pose problems and questions related to everyday routines</td><td><u>Practical Examples</u>
• Use of appropriate number lines / 100 squares to count back
• Using fingers to take some away
• Number rhymes with props
• How many beads are on a longer string than a shorter one?</td></tr>
</table>

RECORDING SUBTRACTION

Year 2	Year 3 (See KS2 Calculation policy also)
Pictures / marks There were 17 bean bags in a bucket. Luke took 9. How many are in the bucket? 	**Pictures / marks** As Year 2 Numbers are often too large for pictures to be efficient but pictures/diagrams will continue to be used where appropriate
Signs and symbols 9 - 4 = □ 9 - □ = 5 □ - 4 = 5 □ - □ = 5 □ = 9 - 4 5 = □ - 4 5 = 9 - □ 5 = □ - □ Extend to 14 + 5 = 20 - □	**Signs and symbols** 19 - 6 = □ 19 - □ = 13 □ - 6 = 13 □ - □ = 13 □ = 19 - 6 13 = □ - 6 13 = 19 - □ 13 = □ - □ Extend to 21 + 6 = 30 - □
Number lines (partly numbered - empty) 43-28 count on  37 - 12 partition  34 – 9 subtract 9  64 – 25 – Count back 20, then 4, then 1 on a numberline	**Number lines** 84 - 56 count on  97 - 25 partition  56 – 19 subtract 19 
Informal jottings 37 - 12 = 37 - 10 - 2 = 27 - 2 = 25 Partition the 2nd number use jottings to solve problems involving subtraction	**Informal jottings** 97 - 25 = 97 - 20 – 5 = 77 - 5 = 72 Partition the second number
Explaining in writing *(Explaining methods and reasoning orally)* Children can find differences in practical situations Use of 100 square to support counting back in tens	**Explaining in writing** 50 - 29 I did 50 take away 30 then added 1.

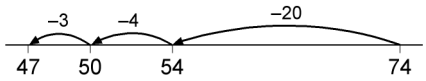
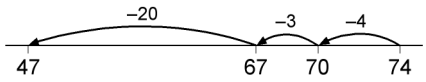
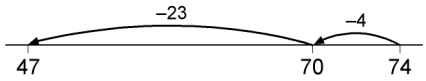
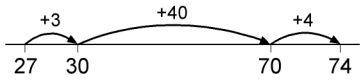
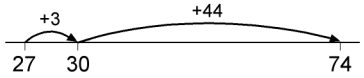
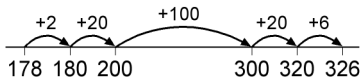
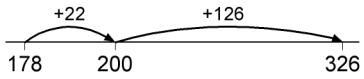
Written methods for subtraction of whole numbers

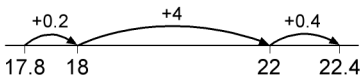
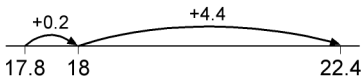
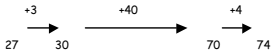
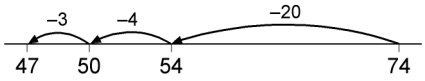
The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for subtraction which they know they can rely on when mental methods are not appropriate.

To subtract successfully, children need to be able to:

- recall all addition and subtraction facts to 20;
- subtract multiples of 10 (such as 160 – 70) using the related subtraction fact,16 – 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.

Stage 1: Using the empty number line	**Stage 1**
Counting back. The empty number line helps to record or explain the steps in mental subtraction. A calculation like 74 – 27 can be recorded by counting back 27 from 74 to reach 47. The empty number line is also a useful way of modelling processes such as bridging through a multiple of ten.	Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10. 15 – 7 = 8  74 – 27 = 47 worked by counting back:  The steps may be recorded in a different order:  or combined: 
The counting-up method (Year 3) • The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47. • With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations such as 57 – 12, 86 – 77 or 43 – 28.	 or: 
Mental method for Year 4 • With three-digit numbers the number of steps can again be reduced, provided that children are able to work out answers to calculations such as 178 + ☐ = 200 and 200 + ☐ = 326 mentally. • The most compact form of recording remains reasonably efficient.	 or: 

• The method can be used with decimals. • This counting-up method can be a useful alternative for children whose progress is slow, whose mental and written calculation skills are weak and whose projected attainment at the end of Key Stage 2 is towards the lower end of level 4.	 or: 
Set out vertically, alongside number line.	Example: 74 - 27 3 (→ 30) 40 (→ 70) 4 (74) 
Stage 2: Partitioning • Subtraction can be recorded using partitioning to write equivalent calculations that can be carried out mentally. For 74 – 27 this involves partitioning the 27 into 20 and 7, and then subtracting from 74 the 20 and the 4 in turn. Some children may need to partition the 74 into 70 + 4 or 60 + 14 to help them carry out the subtraction.	**Stage 2** Subtraction can be recorded using partitioning: 74 – 27 = 74 – 20 – 7 = 54 – 7 = 47 This requires children to subtract a single-digit number or a multiple of 10 from a two-digit number mentally. The method of recording links to counting back on the number line. 

The expanded method for three-digit numbers

Example: 563 – 241, no adjustment or decomposition needed

Expanded method leading to

$$\begin{array}{r} 500+60+3 \\ -\ 200+40+1 \\ \hline 300+20+2 \end{array} \qquad \begin{array}{r} 563 \\ -\ 241 \\ \hline 322 \end{array}$$

Start by subtracting the ones, then the tens, then the hundreds. Refer to subtracting the tens, for example, by saying 'sixty take away forty', not 'six take away four'.

Stage 3

Partitioned numbers are then written under one another:

Example: 74 – 27

	(Year 3)	***(Year 4)***
$70+4$	$\overset{60}{\cancel{70}}+\overset{14}{\cancel{4}}$	$\overset{6}{\cancel{7}}\ \overset{14}{\cancel{4}}$
$-\ 20+7$	$-\ 20+7$	$-\ 2\ 7$
	$40+7$	$4\ 7$

Example: 741 – 367

	(Year 3)	***(Year 4)***
$700+40+1$	$\overset{600}{\cancel{700}}+\overset{130}{\cancel{40}}+\overset{11}{\cancel{1}}$	$\overset{6}{\cancel{7}}\ \overset{13}{\cancel{4}}\ \overset{11}{\cancel{1}}$
$-\ 300+60+7$	$-\ 300+60+7$	$-\ 3\ 6\ 7$
	$300+70+4$	$3\ 7\ 4$

Example: 563 – 271, adjustment from the hundreds to the tens, or partitioning the hundreds

(by end of Year 4)

$500+60+3$	$400+160+3$	$\overset{400}{\cancel{500}}+\overset{160}{\cancel{60}}+3$	$\overset{4}{\cancel{5}}\ \overset{16}{\cancel{6}}\ 3$
$-\ 200+70+1$	$-\ 200+\ 70+1$	$-\ 200+70+1$	$-\ 2\ 7\ 1$
	$200+\ 90+2$	$200+90+2$	$2\ 9\ 2$

Begin by reading aloud the number from which we are subtracting: 'five hundred and sixty-three'. Then discuss the hundreds, tens and ones components of the number, and how 500 + 60 can be partitioned into 400 + 160. The subtraction of the tens becomes '160 minus 70', an application of subtraction of multiples of ten.

Example: 563 – 278, adjustment from the hundreds to the tens and the tens to the ones (Year 5)

$500+60+3$	$400+150+13$	$\overset{400}{\cancel{500}}+\overset{150}{\cancel{50}}+\overset{13}{\cancel{3}}$	$\overset{4}{\cancel{5}}\ \overset{15}{\cancel{6}}\ \overset{13}{\cancel{3}}$
$-\ 200+70+8$	$-\ 200+\ 70+\ 8$	$-\ 200+70+8$	$-\ 2\ 7\ 8$
	$200+\ 80+\ 5$	$200+80+5$	$2\ 8\ 5$

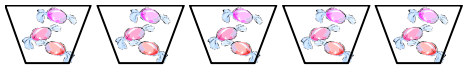
Here both the tens and the ones digits to be subtracted are bigger than both the tens and the ones digits you are subtracting from. Discuss how 60 + 3 is partitioned into 50 + 13, and then how 500 + 50 can be partitioned into 400 + 150, and how this helps when subtracting.

Example: 503 – 278, dealing with zeros when adjusting (by the end of Year 5)

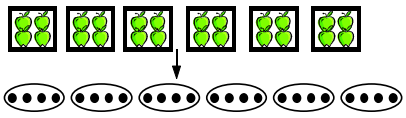
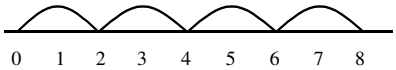
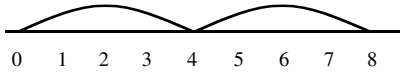
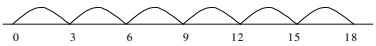
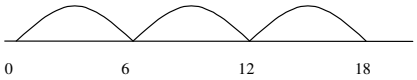
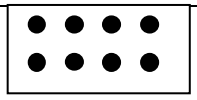
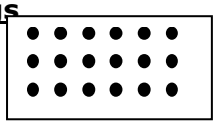
$500+\ 0+3$	$400+90+13$	$\overset{400}{\cancel{400}}+\overset{90}{\cancel{100}}+\overset{13}{3}$	$\overset{4}{\cancel{5}}\ \overset{9}{\cancel{0}}\ \overset{13}{\cancel{3}}$
$-\ 200+70+8$	$-\ 200+70+\ 8$	$-\ 200+70+8$	$-\ 2\ 7\ 8$
	$200+20+\ 5$	$200+20+5$	$2\ 2\ 5$

Here 0 acts as a place holder for the tens. The adjustment has to be done in two stages. First the 500 + 0 is partitioned into 400 + 100 and then the 100 + 3 is partitioned into 90 + 13.

RECORDING MULTIPLICATION

Foundation PRSN – Overviews 5 and 10	Year 1
Pictures / marks How many wheels do we need to make three lego cars? 	**Pictures / marks** There are 3 sweets in one bag. How many sweets are there in 5 bags? 
Signs and symbols	**Signs and symbols**
Number lines Children can move along a numberline Eg. Jumping forward along a number track in ones and twos and fives and tens	**Number lines** (numbered) *(Recording on a number line modelled by the teacher when solving problems)* Link counting in twos, fives and tens to jumping along a number line.
Informal jottings Draw groups of objects in sets	**Informal jottings** Identify missing numbers in number sentences, linked to counting in steps of different sizes
Practical Examples • Pairs of socks in 2s on a washing line • Counting in 2s, 5s and 10s • Count repeated groups of the same size • Sort real objects and pictures into sets of equal number, whilst counting aloud • Show photos of hands on IWB. How can we arrange them to make counting the fingers easier? • Role play opportunities… 'We'll need enough for 6 of us'	**Practical Examples** • Sort objects into groups to count and represent as a picture • Investigating natural multiples by grouping, arranging and sorting… Eggs in a box Corners on squares Fingers and gloves • Children use practical equipment , such as 10ps or straws bundled in tens, to consolidate the count • 'I have three pairs of socks in my basket. How many socks is this?'

RECORDING MULTIPLICATION

<table>
<tr><td>Year 2</td><td>Year 3 (See KS2 calculation policy also)</td></tr>
<tr><td><u>Pictures / marks</u>
There are 4 apples in one box. How many apples in 6 boxes?
  </td><td><u>Pictures / marks</u>
A spider has 8 legs.
How many legs do 4 spiders have?
  </td></tr>
<tr><td>Signs and symbols
□ x 2 = □
6 x □ = 12
□ x 2 = 12
□ x □ = 12
□ = 2 x 6
12 = □ x 6
12 = 2 x □
12 = □ x □
Extend to 4 x 5 = 10 x □</td><td>Signs and symbols
6 x 5 = □
6 x □ = 30
□ x 5 = 30
□ x □ = 30
□ = 5 x 6
30 = □ x 6
30 = 5 x □
30 = □ x □
Extend to 4 x 3 = □ x 2</td></tr>
<tr><td>Number lines (numbered then empty)
2 x 4
 
4 x
 
Recording by - drawing jumps on prepared line
- constructing own lines</td><td>Number lines
3 x 6
 
6 x 3
  </td></tr>
<tr><td><u>Informal jottings</u>
Arrays
2 x 4 or 4 x 2
 
Repeated addition
2 x 4 = 2 + 2 + 2 + 2
Doubling by partitioning
15 x 2 = 30
Partition
10 + 5
20 + 10 = 30</td><td><u>Informal jottings</u>
Arrays
3 x 6 or 6 x 3
 
Repeated addition
3 x 6 = 3 + 3 + 3 + 3 + 3 + 3</td></tr>
<tr><td><u>Practical Examples</u>
• Chanting of tables is supported by counting stick or visual image of a number dial or number line
• Double amounts of money and match each coin with its ‘identical twin’
• Use pegboards to create arrays, where rows and columns are in different colours</td><td></td></tr>
</table>

Written methods for multiplication of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for multiplication, which they know they can rely on when mental methods are not appropriate.

These notes show the stages in building up to using an efficient method for two-digit by one-digit multiplication by the end of Year 4, two-digit by two-digit multiplication by the end of Year 5, and three-digit by two-digit multiplication by the end of Year 6.

To multiply successfully, children need to be able to:

- recall all multiplication facts to 10 × 10;
- partition number into multiples of one hundred, ten and one;
- work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value;
- add two or more single-digit numbers mentally;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- add combinations of whole numbers using the column method (see above).

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.

Stage 1: Mental multiplication using partitioning	**Stage 1**
• Mental methods for multiplying TU × U can be based on the distributive law of multiplication over addition. This allows the tens and ones to be multiplied separately to form partial products. These are then added to find the total product. Either the tens or the ones can be multiplied first but it is more common to start with the tens.	Informal recording in Year 4 might be: 43 40 + 3 ↓ ↓ × 6 240 + 18 = 258 Also record mental multiplication using partitioning: $14 \times 3 = (10 + 4) \times 3$ $= (10 \times 3) + (4 \times 3) = 30 + 12 = 42$ $43 \times 6 = (40 + 3) \times 6$ $= (40 \times 6) + (3 \times 6) = 240 + 18 = 258$ Note: These methods are based on the distributive law. Children should be introduced to the principle of this law (not its name) in Years 2 and 3, for example when they use their knowledge of the 2, 5 and 10 times-tables to work out multiples of 7: ○○○○○○○ ○○○○○…○○ ○○○○○○○ ○○○○○…○○ ○○○○○○○ ○○○○○…○○ $7 \times 3 = (5 + 2) \times 3 = (5 \times 3) + (2 \times 3) = 15 + 6 = 21$ Use arrays to model multiplication, for preparation for the grid method.

Stage 2: The grid method

- As a staging post, an expanded method which uses a grid can be used. This is based on the distributive law and links directly to the mental method. It is an alternative way of recording the same steps.
- It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products.

Stage 2

38 × 7 = (30 × 7) + (8 × 7) = 210 + 56 = 266

×	7
30	210
8	56
	266

Stage 3: Expanded short multiplication

- The next step is to represent the method of recording in a column format, but showing the working. Draw attention to the links with the grid method above.
- Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 38 × 7 is 'thirty multiplied by seven', not 'three times seven', although the relationship 3 × 7 should be stressed.
- Most children should be able to use this expanded method for TU × U **by the end of Year 4.**

Stage 3 **(by the end of Y4)**

```
  38
× 7
 210   30x7
  56   8x7
 266
```

Stage 4: Short multiplication

- The recording is reduced further, with carry digits recorded below the line.

If, after practice, children cannot use the compact method without making errors, they should return to the expanded format or grid method of stage 3.

They could move onto Stage 5 expanded format i.e. grid method TUxTU

Stage 4

```
  38
× 7
 266
 5
```

The step here involves adding 210 and 50 mentally with only the 5 in the 50 recorded. This highlights the need for children to be able to add a multiple of 10 to a two-digit or three-digit number mentally before they reach this stage.

Stage 5: Two-digit by two-digit products

- Extend to TU × TU, asking children to estimate first.
- Start with the grid method. The partial products in each row are added, and then the two sums at the end of each row are added to find the total product.
- As in the grid method for TU × U in stage 4, the first column can become an extra top row as a stepping stone to the method below.

Stage 5

56 × 27 is approximately 60 × 30 = 1800.

×	20	7	
50	1000	350	1350
6	120	42	162
			1512
			1

	50	6	
×	20	7	
	1000	350	1350
	120	42	162
			1512
			1

- Reduce the recording, showing the links to the grid method above.

If at any stage the children are having difficulty then return to the grid method

56 × 27 is approximately 60 × 30 = 1800. (end of Y5)

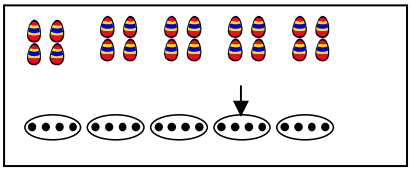
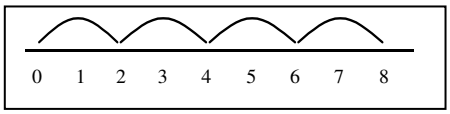
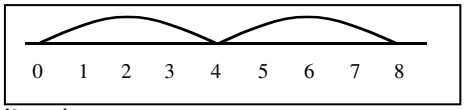
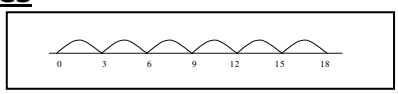
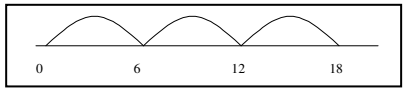
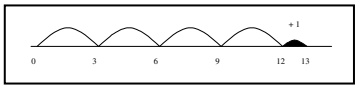
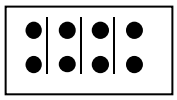
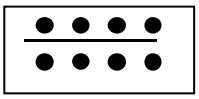
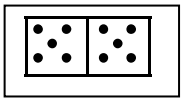
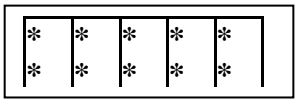
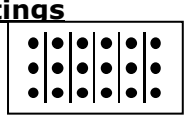
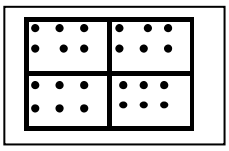
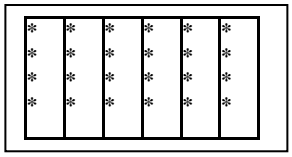
```
  56
× 27
1000   50×20 = 1000
 120    6×20 =  120
 350   50×7  =  350
  42    6×7  =   42
1512
 1
```

• Reduce the recording further. • The carry digits in the partial products of 56 × 20 = 120 and 56 × 7 = 392 are usually carried mentally. • The aim is for most children to use this long multiplication method for TU × TU by the end of Year 5.	56 × 27 is approximately 60 × 30 = 1800. 56 × 27 1120 56×20 392 56× 7 1512 1
Stage 6: Three-digit by two-digit products • Extend to HTU × TU asking children to estimate first. Start with the grid method. • It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products.	**Stage 6** 286 × 29 is approximately 300 × 30 = 9000. × | 20 | 9 | 200 | 4000 | 1800 | 5800 80 | 1600 | 720 | 2320 6 | 120 | 54 | 174 8294 1
• Reduce the recording, showing the links to the grid method above. • This expanded method is cumbersome, with six multiplications and a lengthy addition of numbers with different numbers of digits to be carried out. There is plenty of incentive to move on to a more efficient method.	286 × 29 4000 200×20 = 4000 1600 80×20 = 1600 120 6×20 = 120 1800 200× 9 = 1800 720 80× 9 = 720 54 6× 9 = 54 8294 1
• Children who are already secure with multiplication for TU × U and TU × TU should have little difficulty in using the same method for HTU × TU. • Again, the carry digits in the partial products are usually carried mentally.	286 × 29 is approximately 300 × 30 = 9000. 286 × 29 5720 286×20 2574 286× 9 8294 1

RECORDING DIVISION

Foundation PRSN – Overviews 5 and 10	Year 1
Pictures / marks Grouping How many pairs of socks are there in the 'launderette'? 	**Pictures / marks** 12 children get into teams of 4 to play a game. How many teams are there?  Children record their solution by drawing
Signs and symbols	**Signs and symbols**
Number lines	**Number lines** (numbered) *(Recording on a number line modelled by the teacher when solving problems)*
Informal jottings	**Informal jottings** Solve problems through drawing a diagram Eg. 15 children sit at 3 tables. How many children are at each table if there are the same number at each?
Practical Examples • In the role play area share the place settings between 2 children • Group bags of sweets for the teddies • Follow a recipe:- each child will need 5 cherries	**Practical Examples**

RECORDING DIVISION

Year 2	Year 3 (see KS2 Calculation policy also)
Pictures / marks 4 eggs fit in a box. How many boxes would you need to pack 20 eggs? 	**Pictures / marks** 8 children can travel in a minibus. How minibuses would you need to take 29 children to a football match? 
Signs and symbols 12 ÷ 2 = □ □ = 12 ÷ 2 12 ÷ □ = 6 6 = □ ÷ 2 □ ÷ 2 = 6 6 = 12 ÷ □ □ ÷ □ = 6 6 = □ ÷ □ Extend to 15 -10 = 10 ÷ □	**Signs and symbols** 30 ÷ 5 = □ □ = 30 ÷ 5 30 ÷ □ = 6 6 = □ ÷ 5 □ ÷ 5 = 6 6 = 30 ÷ □ □ ÷ □ = 6 6 = □ ÷ □ Extend to 20 -11 = □ ÷ 5
Number lines (numbered empty) 8 ÷ 2  8 ÷ 4  Recording by - drawing jumps on prepared lines - constructing own lines	**Number lines** 18 ÷ 3  18 ÷ 6  Remainders 13 ÷ 3 
Informal jottings Arrays 8 ÷ 2  or  Sharing 10 ÷ 2 Grouping 10 ÷ 2   Understand division as sharing and grouping	**Informal jottings** Arrays 18 ÷ 3  Sharing 24 ÷ 4 Grouping 24 ÷ 4   Understand division as sharing and grouping

Written methods for division of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for division, which they know they can rely on when mental methods are not appropriate.

These notes show the stages in building up to long division through Years 4 to 6 – first long division TU ÷ U, extending to HTU ÷ U, then HTU ÷ TU, and then short division HTU ÷ U.

To divide successfully in their heads, children need to be able to:

- understand and use the vocabulary of division – for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- recall multiplication and division facts to 10 × 10, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;
- understand and use multiplication and division as inverse operations.

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.

To carry out written methods of division successful, children also need to be able to:

- understand division as repeated subtraction;
- estimate how many times one number divides into another – for example, how many sixes there are in 47, or how many 23s there are in 92;
- multiply a two-digit number by a single-digit number mentally;
- subtract numbers using the column method.

Stage 1: 'Expanded' method for TU÷U and HTU ÷ U	**Stage 1 (the stages build up through year 4 to year 6)**
• This method is based on subtracting multiples of the divisor from the number to be divided, the dividend. • For TU ÷ U there is a link to the mental method. • As you record the division, ask: 'How many nines in 90?' or 'What is 90 divided by 9?' • Once they understand and can apply the method, children should be able to move on from TU ÷ U to HTU ÷ U quite quickly as the principles are the same. • This method, often referred to as 'chunking', is based on subtracting multiples of the divisor, or 'chunks'. Initially children subtract several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to subtract. • Chunking is useful for reminding children of the link between division and repeated subtraction. • However, children need to recognise that chunking is inefficient if too many subtractions have to be carried out. Encourage them to reduce the number of steps and move them on quickly to finding the largest possible multiples.	97 ÷ 9 9)97 − 90 10×9 7 Answer: 10 R 7 6)196 − 60 10×6 136 − 60 10×6 76 − 60 10×6 16 − 12 2×6 4 32 Answer: 32 R 4
• The key to the efficiency of chunking lies in the estimate that is made before the chunking starts. Estimating for HTU ÷ U involves multiplying the divisor by multiples of 10 to find the two multiples that 'trap' the HTU dividend. • Estimating has two purposes when doing a division: – to help to choose a starting point for the division; – to check the answer after the calculation. • Children who have a secure knowledge of multiplication facts and place value should be able to move on quickly to the more efficient recording on the right.	To find 196 ÷ 6, we start by multiplying 6 by 10, 20, 30, … to find that 6 × 30 = 180 and 6 × 40 = 240. The multiples of 180 and 240 trap the number 196. This tells us that the answer to 196 ÷ 6 is between 30 and 40. Start the division by first subtracting 180, leaving 16, and then subtracting the largest possible multiple of 6, which is 12, leaving 4. 6)196 − 180 30×6 16 − 12 2×6 4 32 Answer: 32 R 4 The quotient 32 (with a remainder of 4) lies between 30 and 40, as predicted.

Stage 2: Short division of TU÷U and HTU ÷ U • 'Short' division of TU ÷ U can be introduced as a more compact recording of the mental method of partitioning. 'Short' division of HTU ÷ U can be introduced as an alternative, more compact recording. No chunking is involved since the links are to partitioning, not repeated subtraction. • The accompanying patter is 'How many threes in 290?' (the answer must be a multiple of 10). This gives 90 threes or 270, with 20 remaining. We now ask: 'How many threes in 21?' which has the answer 7. •	**Stage 2** The short division method is recorded like this: $3\overline{)60+21}$ (20 + 7) This is then shortened to: $3\overline{)8\,{}^{2}1}$ (2 7) The carry digit '2' represents the 2 tens that have been exchanged for 20 ones. In the first recording above it is written in front of the 1 to show that 21 is to be divided by 3. In second it is written as a superscript. The 27 written above the line represents the answer: 20 + 7, or 2 tens and 7 ones.
Stage 3: Long division The next step is to tackle HTU ÷ TU, which for most children will be in Year 6. The layout on the right, which links to chunking, is in essence the 'long division' method. Recording the build-up to the quotient on the left of the calculation keeps the links with 'chunking' and reduces the errors that tend to occur with the positioning of the first digit of the quotient. Conventionally the 20, or 2 tens, and the 3 ones forming the answer are recorded above the line, as in the second recording.	**Stage 3** How many packs of 24 can we make from 560 biscuits? Start by multiplying 24 by multiples of 10 to get an estimate. As 24 × 20 = 480 and 24 × 30 = 720, we know the answer lies between 20 and 30 packs. We start by subtracting 480 from 560. $24\overline{)560}$ $-\underline{480}$ 20×24 80 $\underline{72}$ 3×24 8 Answer: 23 R 8 In effect, the recording above is the long division method, though conventionally the digits of the answer are recorded above the line as shown below. $24\overline{)560}$ (23) $-\underline{480}$ 80 $-\underline{72}$ 8 Answer: 23 R 8

Stage 4 Short division of a three-digit number can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound. • **For most children this will be at the end of Year 5 or the beginning of Year 6.**	For 291 ÷ 3, because 3 × 90 = 270 and 3 × 100 = 300, we use 270 and split the dividend of 291 into 270 + 21. Each part is then divided by 3. 291 ÷ 3 = (270 + 21) ÷ 3 = (270 ÷ 3) + (21 ÷ 3) = 90 + 7 = 97 The short division method is recorded like this: $3\overline{)290+1} = \overset{90+\ 7}{3\overline{)270+21}}$ This is then shortened to: $\overset{\ 9\ 7}{3\overline{)2\ 9^{2}1}}$ The carry digit '2' represents the 2 tens that have been exchanged for 20 ones. In the first recording above it is written in front of the 1 to show that a total of 21 ones are to be divided by 3. **The 97 written above the line represents the answer: 90 + 7, or 9 tens and 7 ones.**